

MEETLEARN.COM

Advanced Level Further Mathematics
Paper Three

Order Your Books at

www.shop.meetlearn.com

and receive them
right at your doorstep

June 2003

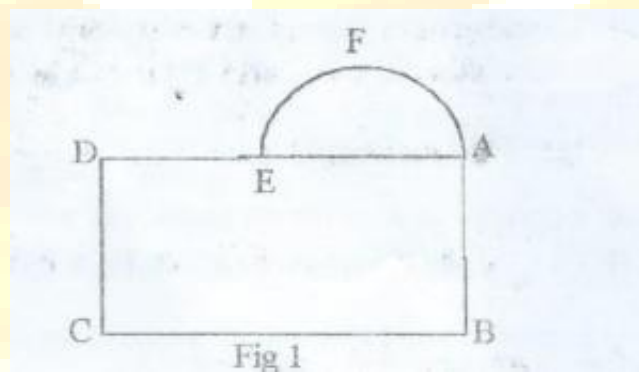
1. At time t seconds, the position vectors $\mathbf{r}_1, \mathbf{r}_2$ of two particles P_1, P_2 are given by $\mathbf{r}_1 = [(t^2 + 1)\mathbf{i} + t^3\mathbf{j} - 6t\mathbf{k}] \text{ m}$, $\mathbf{r}_2 = [(t + 1)2\mathbf{i} + 2(t + 1)\mathbf{j} - 2t(t + 2)\mathbf{k}] \text{ m}$, respectively

a) Find the velocity of P_1 , relative to P_2 when $t = 2$.

b) Show that P_1 , travels in a straight line and find a unit vector parallel to the direction of its motion.

MeetLearn.com

ii) A particle travels in a straight line with an acceleration of $(9 - 4v^2) \text{ ms}^{-2}$, where $v \text{ ms}^{-1}$ is the speed of the particle. The particle starts from rest when $t = 0$. Find the time which the particle will take to attain a speed of 1 ms^{-1} leaving your answer in terms of natural logarithms.



In Fig. 1, A,B,C,D are the vertices of a rectangular Lamina of width $2a$ and length $4a$. E is the midpoint of AD and AFE is a semi-circular lamina with AE as diameter

a) Show, by integration, that the centre of gravity of the semi-circular lamina is at a distance $\frac{4a}{3}$ from AD, along the line of symmetry of the semi-circular lamina.

b) Find the distance from CD and from BC of the centre of gravity of the composite lamina ABCDEF.

The composite lamina is suspended freely from C. Find the tangent of the angle which BC makes with the vertical when the lamina is in equilibrium

3. A uniform ladder of mass m kg rests in limiting equilibrium with

the end A on horizontal

ground and the end B against a vertical wall. The ladder is inclined at $\tan^{-1}(3/4)$ to the horizontal. Given that the coefficient of friction at each end of the ladder is the same, find its value.

A particle of mass $2m$ kg now attached to the midpoint of the ladder and the ladder is adjusted until it rests in limiting equilibrium. Find the tangent of the angle which the ladder now makes with the horizontal.

4. i) A particle of mass 1 kg moves in a straight line under the action of a retarding force which has

Magnitude $\frac{2}{1+t^2}$ N at time t seconds. given that initial speed of the particle is 1 ms⁻¹. Find the time which the particle takes to attain its initial speed. Find also the distance travelled in this time.

ii) A particle of mass 4 kg is suspended from an elastic string of natural length 1.5 m and modulus of elasticity 180 N. The particle is pulled vertically downwards and then released when the length of the string is 2 m. Find its acceleration at the instant that it is released. [Take $g = 10$ ms⁻²]

5.i) A particle of mass 3 kg is attached to one end of a light inextensible string of length 2 m, the other end of which is fixed at a point O. The particle moves in a horizontal circle whose centre is 1.6m vertically below O. Find the tension in the string and the speed of the particle.

[Take g as 10ms^{-2}]

ii) A projectile is fired with initial velocity 5ms^{-1} and angle of projection $\tan^{-1}(4/3)$. The projectile is half way below its maximum height at times t_1 and t_2

show that $t_1 + t_2 = 8u/g$ and $t_1 t_2 = 8u^2/g^2$

6. Two particles P and Q. of masses 5m kg and 2m kg respectively, are connected by a light inextensible string of length 2 m. P is held at rest on a smooth horizontal platform of height 3 m, so that the string is taut and at right angles to the edge of the platform. The particle P is released and when it has travelled a distance a m on the platform, the string breaks. Show

That when P reaches the edge of the platform, Q is at a distance $15a/7$ m below the edge, and that the velocity of Q then is $\frac{11}{4}\sqrt{7ga}\text{ms}^{-1}$ show further that the velocity of Q just before it reaches the bottom of the platform is $\frac{17}{49}\sqrt{7ga}\text{ms}^{-1}$

7. A sphere of mass 2kg and velocity 4ms^{-1} impinges directly on a sphere of mass 4kg and velocity 2ms^{-1} , moving in the same direction. Find, in terms of the coefficient of restitution e ,

- the velocities of the particles after collision,
- the loss in kinetic energy due to the impact, simplifying your answer as much as possible,
- the impulse experienced by the first sphere during the impact.

Find the value of e if the velocity of the second sphere is doubled by the impact. Hence

determine the magnitude of the force exerted on this sphere if the impact lasted for 0.2 s.

8. i) The events A , B and C are independent and $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{2}$ $P(A \cup C) = \frac{1}{2}$ find.

a) $P(A \cup B)$

b) $P(C)$

c) $P(A \cup C)$

Show that the events A , B and C are not collectively exhaustive.

ii) A certain table tennis tournament ends after one player has won two games. Find the probability that all 3 games will be played before the winner in a tournament between two players A and B known, if the probability of winning any match between the two players is $\frac{1}{3}$ for A and $\frac{2}{3}$ for B

