# A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2003 MEETLEARN.COM



*A Level Further Pure Maths*

1. A linear transformation T is represented by the matrix A, where

$$A=\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{pmatrix}.$$

Find

(a) the image of the point $(-5, 4, 9)$ under T.

(b) the image of the line with parametric equations $x = 3t - 1, y = t + 1, z = 1 - 2t$ under T

(c) Given that B is another matrix, where $B=\begin{pmatrix}1 & -3 & 2\\ -3 & 3 & -1\\ 2 & -1 & 0\end{pmatrix}$, find the matrix product AB and BA.

Hence, or otherwise, find the point where the three planes with equations

$$x + 2y + 3z = -5$$
$$2x + 4y + 5z = 3 \text{ intersect.}$$
$$3x + 2y + 6z = 1$$

2. A particular integral of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = \sin x$ is $y = A\cos x + B\sin x$.

Find the values of the constants A and B. Hence, find the general solution of this differential equation. Find, also, the solution of this differential equation for which y = 0 and $\frac{dy}{dx} = 0$ when x = 0

3. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

at the point $P(a\cosh\theta, b\sinh\theta)$.

The tangent meets the asymptotes of the hyperbola at the points Q and R. show that

(a) the coordinates of Q and R are $(ae^{\theta}, be^{\theta})$ and $(ae^{-\theta}, -be^{-\theta})$, respectively.

(b) P is the midpoint of QR

(c) $OQ.OR = a^2 + b^2$, where O is the origin.

This tangent also cuts the x – axis at Sand the y – axis at T. find the area of the triangle SOT.

4. (i) Prove that the set S = {1, 2, 4, 7, 8, 11, 13, 14} forms a group under multiplication modulo 15. There are three subgroups each consisting of 1, 4 and two other elements of S. list these subgroups and prove that two of them are isomorphic.

(ii) Find the value of the real numbers x and y so that the set S = {1, x + iy, x – iy} forms a group under the operation of multiplication of numbers.

5. (i) The function H is defined by $H(x) = 3\cosh\left(\frac{x}{3}\right) + \sinh\left(\frac{x}{3}\right)$.

Find the values of $\lambda$ for which $H(\ln\lambda^3) = 4$

(ii) Show that $\coth^{-1}x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), |x| > 1$ and that $\coth^{-1}(1 + 2\cot^2\alpha) = -\ln|\cos\alpha|, \alpha \neq (2\pi + 1)\frac{\pi}{2}$

Find the values of $\alpha$ between 0 and $2\pi$ which satisfy the equation $2\coth^{-1}(1 + 2\cot^2\alpha) = \ln 2$

(iii) Solve, for x, the equation $x\cosh(\ln x) = 2$

6. (i) Expand $\left(z + \frac{1}{z}\right)^4$ and $\left(z - \frac{1}{z}\right)^4$. By putting $z = \cos\theta + i\sin\theta$, show that

$\cos^4\theta + \sin^4\theta = \frac{1}{4}(\cos 4\theta + 3)$

(ii) (a) Given that $z_1$ and $z_2$ are complex numbers, show geometrically that $|z_1 + z_2| \leq |z_1| + |z_2|$

(b) The complex number z is such that $z^2 + 2z = 8e^{ai}$ where a is real. Show that $|z| > 2$

(iii) Show that the transformation $z \to \omega$, where $\omega = \frac{2z + 4}{iz + 1}$, maps $|z - 1| = |z + 2|$ to $|\omega| = 2$

7. Given that $f(x) = f(n - x)$.

prove that

$$\int_0^{\pi} xf(x)dx = \frac{\pi}{2}\int_0^{\pi} f(x)dx.$$

Hence, evaluate $\int_0^{\pi} \frac{x\sin^3 x}{1+\cos^2 x}dx$

8. (i) Draw the graph of $y = f(x)$, given that $f(x) = |x| + |x+2| - 4$. Hence, or otherwise, find the set of values of x which satisfy the inequality $|x| + |x+2| - 3 \le 0$

(ii) Sketch the curve with polar equation $r = 1 + 2\cos\theta, 0 \le \theta \le 2\pi$

(iii) Given that

$$f(x) = \frac{4x^2 + 8x - 5}{2x - 3},$$

show that $f(x)$ cannot lie between 2 and 18. Sketch the curve $y = f(x)$ showing its asymptotes.

9. (i) The position vectors of the points A, B, C, D, with respect to the origin O, are a, b, c, d respectively, where

(i) Show that,

$\frac{1}{2} \le \frac{x^2 + x + 7}{x^2 - 4x + 5} \le 13\frac{1}{2}$ and sketch the graph of the function

$$f(x) = \frac{x^2 + x + 7}{x^2 - 4x + 5},$$

indicating how the curve approaches its asymptotes.