A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2005 MEETLEARN.COM

A Level Further Pure Maths

1. The points A, B and C have coordinates (3, 2, 4) and (2, 5, 6) and (2, 4, 8) respectively, relative to the Cartesian axes OXYZ. find
   (a) $AB \times AC$
   (b) the area of triangle ABC,
   (c) a vector and Cartesian equation of the plane ABC
   (d) the distance from O to the plane ABC
   (e) the volume of the tetrahedron OABC

2. (a) A curve is given by the parametric equations $x = t^2$, $y = t\left(1 - \frac{t^2}{3}\right)$, $0 \le t \le \sqrt{3}$. Find
   (i) the length of the curve
   (ii) the area of the surface generated when this curve is revolved completely about the x – axis
   (b) Find the mean value of $\tanh^2 hx$ in the interval $0 \le x \le \ln 2$

3. (a) find the general solution of the differential equation $x\frac{dy}{dx} + y = 6x^2$

6. (i) Show that
$$\frac{1 + \tanh^2 3x}{1 - \tanh^2 3x} \equiv \cosh 6x.$$
By using the substitution $t = \tanh 3x$, or, otherwise, find $\int \sec h6x dx$.
(ii) Solve for real x, the equation $e^{\tanh^{-1}x} - 1 = e^{\cosh^{-1}x}$.
(iii) Show that
$$\int_0^{\ln 2} \frac{dx}{5\cosh x - 3\sinh x} = \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\frac{1}{2}$$

7. (i) Use De Moivre's theorem to show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.
Deduce that $\cos^2\frac{\pi}{8} = \frac{2 + \sqrt{2}}{4}$
(ii) Show that, if $f(z) = z^6 - 2z^3\cos 3\alpha + 1$, then $z^3 = 3\cos\alpha \pm i\sin 3\alpha$ if $f(z) = 0$. By using De Moivre's theorem, express $f(z)$ as a product of quadratic factors.

8. (i) Find the length of the arc of the curve $x = \ln t$, $2y = \left(1 + \frac{1}{t}\right)$, between the points (0, 1) and $\left(\ln 2, \frac{5}{4}\right)$.
(ii) Given that $n \in Z^+$ and $I_n = \int_0^{\pi} e^{-x}\sin^n x dx$.
Show that for $n > 1$, $I_n = \frac{n}{1-n}I_{n-2}$

9. (i) sketch, using different axes, the curves with polar equations;
(a) $r = 2 + \sin\theta$
(b) $r = 2\cos 2\theta$
In (b), find the equations of the tangents at the pole.
(ii) Show that, $\frac{1}{2} \le \frac{x^2 + x + 7}{x^2 - 4x + 5} \le 13\frac{1}{2}$, and sketch the graph of the function $f(x) = \frac{x^2 + x + 7}{x^2 - 4x + 5}$, indicating how the curve approaches its asymptotes.