A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2006 MEETLEARN.COM

A Level Further Pure Maths

1. For the curve $y = \frac{1}{6}\left(x^3 + \frac{3}{x}\right)$,

   (a) Show that the length of arc $1 \le x \le 2$ is $\frac{17}{12}$.

   (b) Show also that the area of the surface formed by rotating the arc in (a) through 2π radians about the x - axis is $\frac{47\pi}{16}$.

   (c) Use the theorem of Pappus to find y - coordinate of the centroid of the arc.

2. Given the matrix A, and the line L, where $A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 5 & 2 \\ 1 & 2 & -2 \end{pmatrix}$ and $L: \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-1}{-6}$

   (a) Find the perpendicular distance from the point Q (4, - 1, 3) to the line L.

   (b) Show that A is invertible.

   (c) Find the image of the line L under the transformation with matrix A.

   (d) Find the set of vectors x which are mapped onto themselves under the transformation whose matrix is A.

3. (i) (a) Solve the equation $4\tanh x - \operatorname{sech} x = 1$ leaving your answer in terms of natural logarithms.

   (b) Show that $\int_0^1 \frac{1}{\sqrt{4x^2 + 12x + 10}}\,dx = \frac{1}{2}\ln\left(\frac{5+\sqrt{10}}{3+\sqrt{10}}\right)$.

   (ii) Show that when x is small such that x^4 and higher powers of x can be neglected,

   $\ln(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6}$

4. (i) By using the substitution $u = y^3$, or otherwise, find the general solution of the differential equation $xy^2\frac{dy}{dx} - 2y^3 = x$

   (ii) Determine the values of the constants a and b for which $y = a\cos 2x + b\sin 2x$ is a particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 6\cos 2x$.

   Find the solution of this differential equation for which $y = 0$, $\frac{dy}{dx} = 10$, when x = 0.

6. (i) Show that

$$\frac{1+\tanh^2 3x}{1-\tanh^2 3x} \equiv \cosh 6x.$$

By using the substitution $t = \tanh 3x$, or, otherwise, find $\int \sec h6x dx$.

(ii) Solve for real x, the equation $e^{\sinh^{-1} x} - 1 = e^{\cosh^{-1} x}$.

(iii) Show that

$$\int_0^{\ln 2} \frac{dx}{5\cosh x - 3\sinh x} = \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\frac{1}{2}$$

7. (i) Use De Moivre's theorem to show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.

Deduce that $\cos^2\frac{\pi}{8} = \frac{2+\sqrt{2}}{4}$

(ii) Show that, if $f(z) = z^6 - 2z^3\cos 3\alpha + 1$, then $z^2 = 3\cos\alpha \pm i\sin 3\alpha$ if $f(z) = 0$. By using De Moivre's theorem, express $f(z)$ as a product of quadratic factors.

8. (i) Find the length of the arc of the curve $x = \ln t, 2y = \left(1 + \frac{1}{t}\right)$, between the points (0, 1) and $\left(\ln 2, \frac{5}{4}\right)$.

(ii) Given that $n \in Z^+$ and $I_n = \int_0^{\pi} e^{-x}\sin^n x dx$.

Show that for $n > 1$, $I_n = \frac{n}{1-n} I_{n-2}$

9. (i) sketch, using different axes, the curves with polar equations;

(a) $r = 2 + \sin\theta$ (b) $r = 2\cos 2\theta$

In (b), find the equations of the tangents at the pole.

(ii) Show that, $\frac{1}{2} \le \frac{x^2+x+7}{x^2-4x+5} \le 13\frac{1}{2}$, and sketch the graph of the function $f(x) = \frac{x^2+x+7}{x^2-4x+5}$, indicating how the curve approaches its asymptotes.