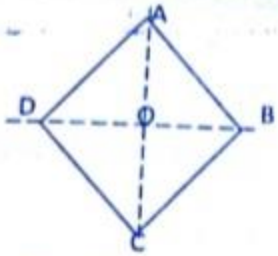
A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2008 MEETLEARN.COM

A Level Further Pure Maths

6. (i)



The figure above shows a rhombus ABCD,. In the set G of symmetry transformation of the rhombus, let

I : denotes the identity transformation,
H: denotes reflection in the diagonal BD
V: denotes reflection in the diagonal AC
R: denotes the rotation of every point through 180° about the center O
Form a combination table for the elements G = {I, H, V, R}. Deduce that these elements form a group.

(ii) Given the matrices $A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}; B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}; C=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$ *and* $D=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$.

Show that the set {A, B, C, D} forms a group under matrix multiplication. State whether or not the group in (i) and (ii) are isomorphic and justify your answer.
[Associativity may be assumed in each case]

7. Prove that the equation of the tangent at the point $(at^2, 2at)$ on the parabola $y^2 = 4ax$ is $x - ty + at^2 = 0$. $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$ are points on this parabola, show that the equation of the chord AB is $2x - (t_1 + t_2)y + 2at_1t_2 = 0$
The tangents at A and B meet at the point P. find the coordinates of P. the line through P parallel to the axis of the parabola meets the chord AB at M. prove that M is the midpoint of AB.

8. (i) In an Argand diagram, shade the region in which the points representing the complex number z can lie if $|z - 1 + 2i| < |z - i|$.

(ii) Use De Moivre's theorem to show that, $\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ and $\sin 5\theta \equiv \cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$, hence, prove that

$$\tan 5\theta = \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}, \text{ where } t = \tan\theta$$

(iii) A transformation T in the complex plane is given by the equation $\omega = \dfrac{iz + 1}{z + i}$.

Show that the circle $|z| = 2$ is mapped onto the circle $|\omega + i| = 2|\omega - i|$

9. (i) Sketch on the same diagram the curves C_1 and C_2, defined by the polar equations

$C_1 : r = 2\sec\theta, -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$C_2 : r = \dfrac{6}{1 + \cos\theta}, -\pi < \theta < \pi$

(a) State the polar coordinates of the points of intersection of C_1 with the initial line.
(b) State the polar coordinates of the points of intersection of C_2 with the initial line and the half lines $\theta = \dfrac{\pi}{2}$ and $\theta = -\dfrac{\pi}{2}$
(c) Find the polar coordinates of the points of intersection of C_1 and C_2.

(ii) Given that $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$, show that, for $n > 2$, $I_n + I_{n-2} = \dfrac{1}{n-1}$