A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2009 MEETLEARN.COM

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A Level Further Pure Maths

1. (i) Find the general solution of the differential equation $x\frac{dy}{dx} - y = x^2$

(ii) Find the value of the constant a such that $y = axe^{-x}$ is a solution of the differential equation

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 2e^{-x}$$

Find the solution of the differential equation for which $y = 1$ and $\frac{dy}{dx} = 2$ when x = 0.

2. (a) Express $f(x) = \frac{1}{(x-1)^2(x^2+1)}$ into partial fractions.

Hence, evaluate $\int_2^3 f(x)dx$.

(b) Given that $I_n = \int_0^{\frac{\pi}{2}} \sin^n \theta d\theta$,

show that for n > 2, $I_n = \left(\frac{n-1}{n}\right)I_{n-2}$. Hence, evaluate I_5

3. (a) Using the definition of $\sinh x$ in terms of e^x, show that $\sinh^{-1} x = \ln\left(x + \sqrt{1+x^2}\right)$.

Hence, or, otherwise show that $\int_0^4 \frac{1}{\sqrt{9x^2+4}}dx \equiv \frac{1}{3}\ln\left(6+\sqrt{37}\right)$

(b) Show that $\int_0^{\frac{1}{4}} \tanh^{-1} 2x dx = \frac{1}{8}\ln\frac{27}{16}$.

4. (i) Determine whether or not the following series converge.

(a) $\sum_{r=0}^{\infty} \frac{r^r}{r!}$ $\left[\text{hint} : \lim_{r\to\infty}\left(1+\frac{1}{r}\right)^r = e\right]$

(b) $\sum_{r=0}^{\infty}\left(\frac{3}{2}\right)^{r^2}$

(c) $\sum_{r=1}^{\infty}\frac{r}{2^r}$

(ii) Given that terms in x^5 and higher powers of x may be neglected,

show that $e^{\cos^2 x} = e\left(1 - x^2 + \frac{6}{5}x^4\right)$

5. (i) Find graphically or otherwise, the values of x for which $|2x-5| + |x+2| > 7$

(ii) $f(x) = \frac{x^2-5x+6}{x-1}$.

(a) Find the equations of the two asymptotes to the curve $y = f(x)$.

(b) Sketch the curve $y = f(x)$, showing clearly the intercepts, asymptotes and turning points.

6. (i) Show that $1+i$ is a root of the equation $z^4 + 2z^3 - z^2 - 2z + 10 = 0$. Hence, or, otherwise, find the remaining roots of the equation.

(ii) If $z = \cos\theta + i\sin\theta$, show that

(a) $z + \frac{1}{z} = 2\cos\theta$

(b) $z^n + \frac{1}{z^n} = 2\cos n\theta$

Hence, or, otherwise show that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$.

Evaluate $\int_0^{\frac{\pi}{2}} \left(32\cos^6\theta - 15\cos 2\theta\right) d\theta$

7. (i) Use the theorem of Pappus to calculate,
(a) The volume and
(b) The surface area, of the solid generated when the region for which $x^2 + (y-9)^2 \le 9$ is rotated through 2π radians.
(ii) A curve is given parametrically by $x = \theta - \sin\theta$; $y = 1 - \cos\theta$
(a) Show that the length of the curve, for $0 \le \theta \le 2\pi$, is 8
(b) If the arc in (a) is rotated through one complete revolution about the x – axis, show that the area of the surface generated is $\frac{64\pi}{3}$

8. (a) Find the equation of the normal at the point P (4, 1) to the rectangular hyperbola $xy = 4$. this normal meets the hyperbola again at the point Q, find the length of PQ.

(b) Write down the equation of the two asymptotes to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

The tangent to this hyperbola at the point $P(3\sec\theta, 4\tan\theta)$ meets the asymptotes at S and T. show that p is the mid – point of ST.

9. (i) A binary operation $*$ is defined on $\Re$, the set of real numbers, by $a * b = a + b + ab$
Determine whether or not $(\Re, *)$ is a group.
(ii) Define a mapping f from $(\Re, \times)$ to $(\Re, +)$, where $\Re$ is the set of integers.
(a) Show that f is a homomorphism
(b) Show, also, that f is an isomorphism.