A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2010 MEETLEARN.COM

A Level Further Pure Maths

1. (i) solve the differential equation

$$(x+2)\frac{dy}{dx}-y=(x+2)^3,$$ given that when x = 0 , y = - 4

(ii) Given that $y=Axe^x+Bxe^{2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^x+2e^{2x},$$ find the constants A and B

Hence, solve completely the differential equation given that $y=0$, $\frac{dy}{dx}=1$ when x = 0.

2. The position vectors of the points A, B, C and D with respect to the origin O are a, b, c, and d respectively, where $a=7i+2j+k$, $b=i-3j+5k$, $c=i+j-4k$ and $d=2i-j+3k$.

Find

(a) the Cartesian equation of the plane ABC;

(b) the Cartesian equation of the plane BCD;

(c) a cosine of the acute angle between the planes ABC and BCD;

(d) the area of the triangle BCD

(e) the volume of the tetrahedron ABCD.

3. Prove that the equation of the normal to the rectangular hyperbola $xy=c^2$ at the point $P\left(ct,\frac{c}{t}\right)$ is $t^3x-ty=c\left(t^4-1\right)$

The normal at P on the hyperbola meets the x – axis at Q and the tangent t P meets the y – axis at R.

Show that the locus of the mid-point of QR, as P varies is $2c^2xy+y^4=c^4$.

4. (a) Find the root mean square value of $\tanh x$ for $0\le x\le 2$

(b) A curve is given parametrically by $x=\cosh^2 t$, $y=2\sinh t$, $0\le t\le 2$.

i. Find the length of the curve, leaving your answer in term of e.

ii. Prove that the area of the surface generated by rotating the curve through 2π radians about the x-axis is given by $\frac{\pi}{3e^6}\left[\left(e^4+1\right)^3-8e^6\right]$

5. (a) Prove that the set of numbers {1, 2, 4, 5, 7, 8}forms an Abelian group under multiplication modulo 9.

(b) Prove also that the set of numbers {1, 2, 4, 5} forms an Abelian group under addition modulo 6. Are the two groups isomorphic? Give a reason to justify your answer.

6. (a) Test each of the following series for convergence

(i) $\sum_{n=0}^{\infty}\left(\frac{2^n+5}{3^n}\right)$

(ii) $\sum_{n=1}^{\infty}\frac{n-1}{2n^2(n+1)}$

(iii) $\sum_{n=4}^{\infty}\frac{\sqrt{n}}{n-3}$

(b) Find the first three terms in the Taylor series expansion of $\tan x$ in ascending powers of $\left(x-\frac{\pi}{4}\right)$.

Deduce that if $\left(x-\frac{\pi}{4}\right)$ is so small that $\left(x-\frac{\pi}{4}\right)^2$ and higher powers may be neglected, then $\tan x=1-\frac{\pi}{2}+2x$.

7. (a) Given that Z_1 and Z_2 are complex numbers, show geometrically, or, otherwise that $|z_1|-|z_2|\le|z_1-z_2|$.

Hence, or, otherwise, show that if Z is a number such that $\left|z^2-3z\right|=4e^{i\alpha}$, where α is real, then $|z|\le 4$

(b) Given that $z=e^{i\theta}$, show that $\frac{\cos 5\theta}{\cos\theta}=16\sin^4\theta-12\sin^2\theta+1$, for $\cos\theta\ne 0$.

(c) Show that the transformation $\omega=\frac{3z+6i}{iz-1}$ maps the line $|z+i|=|z+2i|$ to the curve $|\omega|=3$

8. (a) Solve for real x, the equation $\sinh 2x-2\cosh 2x+2=0$.

(b) Given that $z = e^{i\theta}$, show that $\dfrac{\cos 5\theta}{\cos\theta} = 16\sin^4\theta - 12\sin^2\theta + 1$, *for* $\cos\theta \neq 0$.

(c) Show that the transformation $\omega = \dfrac{3z + 6i}{iz - 1}$ maps the line $|z + i| = |z + 2i|$ to the curve $|\omega| = 3$

8. (a) Solve for real x, the equation $\sinh 2x - 2\cosh 2x + 2 = 0$.

(b) Express $\tanh x$ in terms of e^x and e^{-x} and hence show that $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$.

(c) Given that $I_n = \int_0^{\frac{\pi}{4}} \tanh^n x\,dx$, show that $I_n + I_{n-2} = \dfrac{1}{n-1}\,(n \geq 2)$.

Hence, find $\int_0^{\frac{\pi}{8}} \tanh^n(2x)\,dx$

9. (a) Prove that if A and B are $n \times n$ non - singular matrices, then $(AB)^{-1} = B^{-1}A^{-1}$.

(b) Show that under the transformation, represented by the matrix M, where

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & -2 \\ 3 & -2 & -7 \end{pmatrix},$$

the whole space is mapped onto the plane $x - 2y + z = 0$.

Find the image under this transformation of

(i) the line $x = -y = \dfrac{z-1}{2}$,

(ii) the plane $x - y - z = 0$, giving your answer in Cartesian form.