A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2015 MEETLEARN.COM

A Level Further Pure Maths

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1. Express $8\cosh(x+\ln 4)$ in the form $Ae^{x}+Be^{-x}$ where A and B are real numbers.
   Hence, or otherwise,
   (i) find the two solutions of the equation $8\cosh(x+\ln 4)=12-e^{-x}$
   (ii) show that
   $$\int_{0}^{\ln 2}\frac{1}{\cosh(x+\ln 4)}dx=2\tan^{-1}\left(\frac{4}{33}\right)$$

2. (a) Find the set of values of x for which
   $$\sum_{r=1}^{\infty}(-1)^{r-1}\frac{x^{r}}{2^{r}(r+1)}$$ is convergent
   (b) The function f, for each positive integer n, is given by $f(n)=15^{n}-8^{n-2}$.
   (i) Express $f(n+1)-8f(n)$ in the form $k(15^{n}), k\in Z$
   (ii) Hence, or otherwise, prove that $15^{n}-8^{n-2}$ is a multiple of 7 for all $n\geq 2$

3. (a) Solve completely the equation
   $z^{6}-64=0$, giving your answers in the form $re^{i\theta}, r>0, -\pi<\theta\leq\pi$
   (b) Given that $\omega=e^{i\theta}, \theta\neq n\pi, \pi\in$ [illegible], show that
   i. $$\frac{\omega^{2}-1}{\omega}=2i\sin\theta$$
   ii. $$(1+\omega)^{2}=2^{n}\left(\frac{1}{2}\theta\right)e^{\frac{1}{2}(i\theta)}$$
   (c) Given that one of the roots of the equation $z^{4}=a(1+i\sqrt{3})$ is $z=2e^{\frac{1}{12}i}$. Find the value of a

4. By using the substitution $\frac{dy}{dx}=v-x$, or otherwise, show that the differential equation
   $$(x^{2}-1)\frac{d^{2}y}{dx^{2}}-2x\frac{dy}{dx}=x^{2}+1$$
   can be transformed to the differential equation
   $$\frac{dv}{dx}=\frac{2xv}{x^{2}-1}, \text{ where } v=f(x).$$
   Hence find the solution, in the form $y=f(x)$, of the differential equation
   $$(x^{2}-1)\frac{d^{2}y}{dx^{2}}-2x\frac{dy}{dx}=x^{2}+1, \text{ for which } y=-2, \frac{dy}{dx}=1, \text{ when } x=2$$

5. (a) The parametric equations of a curve are

$x = \frac{2}{3}(t-1)^{\frac{3}{2}}$ *and* $y = \frac{2\sqrt{2}}{3}(1+t)^{\frac{3}{2}}$, $2 \le t \le 4$

Show that the length of the arc is

$$\frac{4\sqrt{3}}{3}\left(4-\sqrt{2}\right)$$

(b) A function f is given by

$$f(x) = \frac{1}{4}\sqrt{4x-1}, \text{ for } 1 \le x \le 9.$$

Show that the surface area obtained when the curve $y = f(x)$ is rotated completely about the x - axis is $\frac{104}{3}\pi$

6. Find the equation of the normal at the point $P(4\cos\theta, 3\sin\theta)$ on the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

The normal at P crosses the x - and y - axes at A and B respectively.

(i) Find the coordinates od A and B.

(ii) Show that as θ varies, the locus of the point M, the midpoint of AB is another ellipse and find its equation.

7. (a) Investigate and sketch the curve $y = |x-2| + |3x+4|$.

Hence, or otherwise, solve the inequality $|x-2| + |3x+4| \le 6$

(b) Given that $r = 2a\sin^2\theta, a > 0$ is the polar equation of a curve.

(i) Find the equation of the tangents at the pole to the curve.

(ii) Show that $f(\theta) = 2a\sin^2\theta$ is an even function.

(iii) Sketch the curve $r = 2a\sin^2\theta$.

8. (a) Two plane Π_1 *and* Π_2 with equations $px + 4y - 2z = 10$ *and* $5x + y + pz = 13$ respectively are perpendicular, find the value of p.

(b) Two lines L_1 *and* L_2 are such that $L_1 : r = 2i + j - k + \lambda(i + j + k)$ *and* L_2 passes through the point $(3, 1, -1)$ and is parallel to the line $r = j + t(2i + j + k)$.

Find

(i) the position vector of the intersection, r_0, of L_1 *and* L_2.

(ii) the Cartesian equation of the plane containing L_1 *and* L_2.

(iii) The area of the triangle with vertices at the point with position vector r_0 and the points when $\lambda = 2$ on L_1 and $t = 1$ on L_2

9. (a) Given that x, y are integers, find the general solution of the

9. (a) Given that x, y are integers, find the general solution of the equation

$$969x - 1683y = 51$$

(b) Using Euclid's algorithm, or otherwise, find the greatest common divisor, h, of the numbers $a = 1625$ *and* $b = 858$ and express it in the form, $h = sa + tb$, where s *and* t are integers