A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2016 MEETLEARN.COM

A Level Further Pure Maths

1. Given that $y = Axe^{2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 4y = 8e^{2x}$$

Find

(i) the value of the constant A.

(ii) the general solution of the differential equation.

(iii) a particular solution of the differential equation for which $y = 1, \frac{dy}{dx} = -8, when\ x = 0$

2. The position vectors of the points A, B and C are $(-i + j + 2k), (2i + 3j + 3k)$ *and* $(-i - 3j)$ respectively, relative to the origin.

Find

(i) the Cartesian equation of the plane ABC.

(ii) the distance of the point $D(-2, -2, 5)$ from ABC.

3. (a) Determine whether or not the set of vectors $v_1 = (2, -2, 4), v_2 = (3, -5, 4)$ *and* $v_3 = (0, 1, 1)$ are linearly dependent.

(b) Determine the null space of the matrix

$$M = \begin{pmatrix} 1 & -7 \\ -3 & 21 \end{pmatrix}.$$

4. (a) The polar curve C, has equation $C: r = \sqrt{2} + 2\sin\theta$.

Find the coordinates of the point that is furthest from the pole

Find the tangents to the curve at the pole and sketch the curve

(b) Given the function f defined by $f(x) = x - 1 + \frac{1}{e^x}$

(i) Evaluate $\lim_{x \to -\infty} f(x)$.

(ii) Evaluate $\lim_{x \to +\infty} [f(x) - (x - 1)]$ and deduce the asymptote to the curve $y = f(x)$.

(iii) Investigate the relative position of the curve to its asymptotes and the variation of the curve considering the monotonicity of f and present the information on a table.

(iv) Sketch the curve $y = f(x)$

5. (a) Given the sequence U_n defined by

$$U_1 = 3,\ U_2 = 5\ and\ U_{n+2} - 3U_{n+1} + 2U_n = 0$$

Prove by mathematical induction that $U_n = 2^n + 1$

(b) Use De Moivre's theorem to show that

$$\sum_{r=1}^{n} \cos rx = \frac{1}{2}$$

6. (a) Evaluate $I = \int_{-1}^{0} \frac{dx}{\sqrt{x^2 + x + 1}}$ giving your answer in the form $\ln a, a > 0$.

6. (a) Evaluate $I = \int_{-1}^{0} \frac{dx}{\sqrt{x^2 + x + 1}}$ giving your answer in the form $\ln a, a > 0$.

(b) Given the function f defined by $f(x) = \frac{|x-2|}{1-|x|}$

(i) State the domain of f.

(ii) Show that

$$f(x) = \begin{cases} \frac{2-x}{1+x}, x < 0 \\ \frac{2-x}{1-x}, 0 \le x < 2 \\ \frac{x-2}{1-x}, x \ge 2 \end{cases}$$

(iii) Investigate the continuity of f at $x = 2$

7. (a) Find the greatest common divisor, d, of the integers 770 and 112. Express d in the form $d = 770x + 112y, x, y \in \mathbb{Z}$

Hence, find the general solution of the equation $770x + 112y = 28$.

(b) Prove by contradiction that if n is even then n^2 is even

8. (a) Given that m and n are positive integers and that

$I_{m,n} = \int_{-\frac{1}{2}}^{\frac{1}{2}} (1-2x)^m (1+2x)^n \, dx$. Show that $I_{m,n} = \left(\frac{m}{n+1}\right) I_{m-1,n+1}, m \ge 1, n \ge 0$ Evaluate $I_{2,4}$.

(b) Given that $f(x) = \frac{2x^2 + 2x + 8}{x^2 + 4}$

Show that $f(x) = a + \frac{2x}{x^2 + 4}$ and find the value of a

Show further that the point $(0, 2)$ is the centre of symmetry of the curve $y = f(x)$.

Given that $g(x) = f(x) - e^{-x}$, evaluate $\lim_{x \to -\infty} g(x)$

9. (a) Given the function

$f(x) = x^7 - x^3 + \frac{\tan x}{(1+x^4)^2}$, determine the parity of f, and hence or otherwise,

evaluate $\int_{-1}^{1} f(x) dx$

(b) Given that $(2Z, +)$ and $(3Z, +)$ are two groups and that $f : 2Z \to 3Z$ is a mapping defined as

$f(x) = \frac{3}{2}x$