A LEVEL FURTHER PURE MATHEMATICS (PAPER 2) 2017 MEETLEARN.COM

A Level Further Pure Maths

Hence, show that $\int_0^1 \frac{5x^2}{(x^2-4)(x^2+1)}dx = \frac{\pi}{4} - \ln 3$

3) A sequence (U_n) is defined recursively by

$$U_0 = \frac{1}{2} \text{ and } U_{n+1} = \frac{2}{1+U_n}, \text{ for all } n \in \mathbb{N}$$

a) Find U_1 and U_2

b) Show by mathematical induction that all the terms of the sequence are positive.

c) Given that the sequence (U_n) is convergent, show that its limit, l, is a solution of the equation $x^2 + x - 2 = 0$. Hence find l

d) Given that (V_n), a sequence of general term such that

$$V_n = \frac{U_n - 1}{U_n + 2}, \forall n \in \mathbb{N}$$

Show that (V_n) is convergence and determine its limit.

Hence, deduce the convergence of the sequence (U_n)

4) (i) Show that

$$\frac{d}{dx}\left(2\tan^{-1}e^x\right) = \sec hx$$

(ii) A curve is given by the parametric equations

$$x = \sec ht, y = \tanh t, \quad 0 \le t \le \frac{1}{2}\ln 3.$$

Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sec h^2 t$

(iii) Show that the length of the curve is $\frac{\pi}{6}$

(iv) The curve is rotated through 2π radians about the $x-axis$. Show that the area of the surface generated is $\pi(2-\sqrt{3})$

5) (a) Using truth tables, or otherwise, determine which of the compound statements $(P \wedge \sim P) \Rightarrow Q$, and $(P \wedge \sim P) \vee Q$ is a tautology and which is a contingency

(b) Prove that an integer is odd if and only if n^2 is odd.

6) A transformation M is given by

$$M = \begin{pmatrix} 3 & 2 & 2 \\ -1 & 0 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

i. Find the inverse of M

ii. Find the values of λ for which $|M - \lambda I| = 0$, where I is the identity matrix.

iii. Find the image of the plane $x + 3y = 0$ under the transformation M

7) Given that $z = \cos\theta + i\sin\theta$,

Show that $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ and find $\sin 5\theta$.

Hence, or otherwise, show that

$$\cot 5\theta = \frac{\cot^5\theta - 10\cot^3\theta + 5\cot\theta}{5\cot^4\theta - 10\cot^2\theta + 1}$$

Solve completely the equation $\cot 5\theta = 0$, show that $\cot\left(\frac{\pi}{10}\right) = \sqrt{5+\sqrt{20}}$

8) (a) (i) Given congruence $7^n \equiv k(\text{mod}\,9)$, find k for each of the values $n = 1,2,3,4$

(ii) Hence, or otherwise find the remainder when $7^{[illegible]}$ is divisible by 9

(iii) Find the general solution of the congruence $7^x \equiv 1(\text{mod}\,9)$, $x \geq 0$

(b) A set F under function composition, $\circ$, has elements $\{f_1, f_2\}$, where $f_1 = x$, $f_2 = \frac{1}{x}$.

Show that $(F, \circ)$ is a group

9) (a) Show that the vectors

$v_1 = \begin{pmatrix}1\\1\\2\end{pmatrix}, v_2 = \begin{pmatrix}0\\-1\\1\end{pmatrix}, v_3 = \begin{pmatrix}2\\1\\3\end{pmatrix}$ are linearly independent

Express $\begin{pmatrix}1\\0\\0\end{pmatrix}$ as a linear combination of v_1, v_2 *and* v_3

(b) Find the Cartesian equation of an ellipse with foci at the points $(0,0)$ *and* $(8,0)$ and eccentricity $\frac{1}{2}$

10) Given that $f(x) = \ln(1+\cosh 2x)$

i. State the domain of f

ii. Show that as $x \to \infty$, $f(x) \to 2x - \ln 2$

iii. Find the asymptotes to the curve $C: y = f(x)$

iv. Investigate the variation of f and draw the table of variation.

v. Sketch C