CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**Paper 3**
Subject Code No.	**0775**

Two and a half hours

Answer ALL questions.

For your guidance the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Turn Over

1. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act through the points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively, where

$$\mathbf{F}_1 = (3i - 2j - 4k)N, \quad \mathbf{r}_1 = (i + k)m$$
$$\mathbf{F}_2 = (-i + j)N, \quad \mathbf{r}_2 = (j + k)m$$
$$\mathbf{F}_3 = (-i + 4k)N, \quad \mathbf{r}_3 = (i + j + k)m$$

(i) Show that this system does not reduce to a single force. **(6 marks)**

When a fourth force F is added, the system of four forces is in equilibrium.

(ii) Show that F acts through the point with position vector $(3k)m$. **(6 marks)**

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2. (a) The equation of motion of a particle P moving in a straight line OX is

$$3\frac{(d^2x)}{(dt^2)} + 6\frac{dx}{dt} + 4x = 0,$$ where x is the displacement of P from O at time t.

Initially, P is at O, moving with speed $\sqrt{3}\,ms^{-1}$.

(i) Show that the displacement x of P can be written in the form $x = Ae^{-t}cos(nt + \varepsilon)$, stating the values of A, n and ε. **(7 marks)**

(ii) Find the period of the motion. **(2 marks)**

(b) A particle performs simple harmonic motion with centre O and amplitude 2 metres. The period of oscillation is π seconds. P and Q are two points which lie at a distance $\sqrt{3}\,m$ on either side of O.

Find the time taken by the particle to move directly from P to Q. **(4 marks)**

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3. Given that

$$\frac{dy}{dx} + x^2 - ylnx = 0$$ and that $y = 0$ when $x = 1$, find the first three non – zero terms in the Taylor series expansion of y for values of x close to 1. **(6 marks)**

(i) Find the value of y when $x = 0.9$. **(2 marks)**

Hence, or otherwise, use the approximation

$$2h\left(\frac{dy}{dx}\right)_n \cong y_{n+1} - y_{n-1}$$ and a step length of 0.1 to find

(ii) the value of y when $x = 1.3$, giving your final answer correct to 4 decimal places. **(6 marks)**

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4. A smooth sphere P moves on a horizontal table and collides with an identical sphere Q at rest. At impact, the direction of motion of P makes an angle of 45^0 with the line of centres of the spheres. Given that the coefficient of restitution between the spheres is e and that after impact, the direction of motion of P makes an acute angle θ with the line of centres of the spheres, show that

(i) $tan\theta = \left(\frac{2}{1-e}\right)$ **(7 marks)**

(ii) $0 < cot\theta \leq \frac{1}{2}$ **(2 marks)**

Given that each sphere is of mass m and that the speed of P before impact is U,

(iii) find the loss in kinetic energy due to the impact. **(3 marks)**

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5. A particle moves round the polar curve

$$r = a\left(2 + \cos\theta\right),\ a > 0,$$

with constant angular velocity ω.

(i) Find, in terms of r, a and ω, the radial component of the acceleration of the particle. **(4 marks)**

(ii) Show that the maximum magnitude of the acceleration of the particle is $4a\omega^2$, stating the angle at which this occurs. **(5 marks)**

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6. A uniform circular disc of mass m and radius $2a$, centre O, is smoothly pivoted at a point A, where OA$=a$.

(i) Find the moment of inertia of the disc about an axis through A perpendicular to the plane of the disc. **(2 marks)**

The disc is free to rotate in a vertical plane about the axis through A. Given that the disc is held with O directly above A and then slightly displaced so that it swings in a vertical plane,

(ii) show that in the ensuing motion,

$$3a\left(\frac{d\theta}{dt}\right)^2 = 2g(1 - cos\theta),$$

where θ is the angle AO makes with the upward vertical. **(3 marks)**

(iii) Show further that when the disc has rotated such that AO makes an angle θ with the upward vertical, where $\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$,

$$t = \frac{1}{2}\sqrt{\frac{3a}{g}}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{cosec}\left(\frac{\theta}{2}\right)d\theta.$$ **(2 marks)**

(iv) Find the reaction at the pivot when $\theta = \pi$. **(6 marks)**

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7. A particle of mass m is projected vertically downward from a great height with speed $\frac{g}{3k}$ in a medium whose resistance to motion is mkv, where v is the speed at time t and k is a constant. The speed doubles after time T when the particle has fallen a distance X.

Show that

(i) $kT = \ln 2$. **(6 marks)**

(ii) $k^2X = g\left(\ln 2 - \frac{1}{3}\right)$. **(7 marks)**

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8. (a) A discrete random variable X has probability mass function defined by

$$f(x) = \begin{cases} c\left(10 - x^2\right), \text{ for } x = 1, 2 \\ c\left(18 - x^2\right), \text{ for } x = 3, 4 \\ 0, \text{ otherwise} \end{cases}$$

Find

(i) the value of the constant c, **(2 marks)**

(ii) the median of X, **(2 marks)**

(iii) the mean and standard deviation of X. **(5 marks)**

(b) A continuous random variable Y has normal distribution with mean 4 and variance 1. Given that $P(Y > k) = 0.025$, determine the value of k. **(5 marks)**

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