Pure Maths With Statistics 2
0770/2

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Statistics**
Paper No.	**Paper 2**
Subject Code No.	**0770**

Three hours

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page

1. The polynomial function $f(x) = 2x^3 + px^2 + qx - 30$ leaves a remainder -28 when divided by $(x - 1)$ and a remainder 66 when divided by $(x - 3)$.

   (a) Find the values of the constants p and q. **(5 marks)**

   (b) Given that $x - 2$ is a factor of $f(x)$, factorise $f(x)$ completely. **(5 marks)**

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2. (i) The roots of the quadratic equation $2x^2 - x + 6 = 0$ are α and β. Find the quadratic equation with integral coefficients whose roots are $\alpha - 2\beta$ and $\beta - 2\alpha$. **(7 marks)**

   (ii) Find the value of the constant k for which the quadratic equation $x^2 + (2 - k)x + 2(2 - k) = 0$ has imaginary roots. **(5 marks)**

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3. (i) Express $\frac{x-1}{(x+2)(x+1)}$ in partial fractions. **(3 marks)**

   (ii) Of Peter's 13 friends, 7 are older than him. In how many ways can he invite 6 friends including at least 4 older friends? **(5 marks)**

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4. Express $\cos 3\theta - \sqrt{3}\sin 3\theta$ in the form $R\cos(3\theta + \beta)$, where $R > 0$ and β is an acute angle.

   Hence, or otherwise, find the general solution of the equation

   $$\cos 3\theta - \sqrt{3}\sin 3\theta = \sqrt{2}.$$

   Determine the minimum and maximum values $\frac{5}{2\cos 3\theta - 2\sqrt{3}\sin 3\theta + 7}$. **(11 marks)**

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5. (i) Given that $\frac{z+1}{z-1} = i$, express z in the form $a + bi$, where a and b are real constants.

   Hence, find $|2z|$. **(5 marks)**

   (ii) The ninth term of an arithmetic progression is *three times* the third term. If the sum of the first four terms is 30, find the first term and common difference of the progression. **(6 marks)**

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6. Find $\frac{dy}{dx}$ if

   (a) $y = \frac{x^2-1}{x^2+1}$, **(3 marks)**

   (b) $y = \cos^4 x$, **(2 marks)**

   (c) $x^2 + y^2 = 16x$. **(3 marks)**

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7. Evaluate

(a) $\int_1^3 \frac{x^2}{x+1}\,dx,$ **(6 marks)**

(b) $\int_0^\pi e^{\cos x} \sin x\ dx.$ **(3 marks)**

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8. (i) Given the statements

p : John is sick,

q : John will go to school,

translate into ordinary English the statements,

(a) $\sim p \wedge q,$

(b) $\sim (p \wedge \sim q)$ **(2 marks)**

(ii) A relation R is defined on the set of ordered pairs of real numbers by

$$(a, b)R(c, d) \text{ if and only if } a^2 + b^2 = c^2 + d^2.$$

Prove that R is an equivalence relation. **(7 marks)**

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9. (i) The vector equation of two lines L_1 and L_2 are given by

L_1 : $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k}),$

L_2: $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + a\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant.

Find the value of a if L_1 and L_2 intersect and the position vector of the point of intersection. **(5 marks)**

(ii) The matrices $\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \\ 2 & -1 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 3 & 2 \\ 2 & 0 & 1 \end{pmatrix}$.

Calculate the matrix product **AB.**

Hence find $\mathbf{B}^{-1}\mathbf{A}^{-1}$ **(8 marks)**

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10. (i) A function $f(x) = \begin{cases} x^2 - 3 & \text{for } 0 \le x < 2, \\ 4x - 7 & \text{for } 2 \le x < 4 \end{cases}$ is such that $f(x) = f(x + 4)$.

Find, $f(27)$ and $f(-106)$. **(3 marks)**

(ii) Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + xy = 3y$ given that $y = 1$ and $x = 0$,

expressing the result in the form $y = f(x)$. **(8 marks)**
