CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics with Mechanics**
Paper No.	**Paper 3**
Subject Code No.	**0765**

Three hours

Full marks may be obtained for answers to ALL questions.

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae booklet published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

1. The position vector of a particle P, of mass 6 kg, at time t seconds is $\mathbf{r}$, where

$$\mathbf{r} = [(18t - 4t^3)\mathbf{i} + t^2\mathbf{j}] \text{ m}.$$

(a) Find the kinetic energy of P when $t = 1$. **(5 marks)**

(b) Calculate the magnitude of the force acting on P when $t = 2$. **(5 marks)**

(c) Determine, to 2 decimal places, the value of t for which the velocity and the acceleration vectors of P are at right angles. **(3 marks)**

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2. A smooth sphere A, of mass $3m$, moving on a smooth horizontal table with speed $4u$, impinges directly on another smooth sphere B, of mass $2m$, moving with speed u in the opposite direction to A. The coefficient of restitution between A and B is e.

(a) Find, in terms of e and u, the speed of B after the impact. **(6 marks)**

If immediately after the collision the speed of A is $\frac{4u}{3}$,

(b) show that $e = \frac{1}{3}$, **(2 marks)**

(c) find the impulse exerted on A by the impact. **(2 marks)**

At the moment of impact, the line of centres of the spheres is perpendicular to a vertical wall which is at a distance x from the point of collision and nearer to B than to A, and B subsequently collides with the wall.

(d) Find, in terms of x, the distance of A from the wall at the instant when B hits the wall. **(3 marks)**

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3. A golf player hits a ball from a point O on horizontal ground with a velocity of $v\sqrt{(13)}$ m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{3}{2}$. The ball first hits the ground at a point A, where $OA = 120$ m.

(a) Find the value of v. **(3 marks)**

(b) Show that its height h above the ground at time t seconds is given by

$$h = (30t - 5t^2) \text{ m}.$$ **(3 marks)**

Hence, or otherwise, find

(c) the maximum height reached by the ball above the level of projection. **(3 marks)**

(d) the time of flight, **(1 mark)**

(e) the Cartesian equation of the trajectory of the ball. **(3 marks)**

(Take g as 10 m s^{-2}.)

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4. (i) The forces $\mathbf{F}_1 = (2\mathbf{i} + b\mathbf{j})$ N, $\mathbf{F}_2 = (-\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} - 4\mathbf{j})$ N act through the points with position vectors $\mathbf{r}_1 = (\mathbf{i} + 3\mathbf{j})$ m, $\mathbf{r}_2 = (x\mathbf{i} + 5\mathbf{j})$ m and $\mathbf{r}_3 = (-\mathbf{i} + \mathbf{j})$ m respectively.

Given that this system of forces is equivalent to a couple of magnitude 12 N m, find

(a) the values of the scalars a and b, **(4 marks)**

(b) the possible values of the scalar x. **(5 marks)**

(ii) A particle starts from rest and moves in a straight line on a smooth horizontal surface. Its acceleration at time t seconds is $k(4v + 1)$ m s^{-2}, where k is a positive constant and v ms^{-1} is the speed of the particle. Given that $v = \frac{e^2 - 1}{4}$ when $t = 1$, show that

$$v = \frac{1}{4}(e^{2t} - 1).$$ **(4 marks)**

(Take g as 10 m s^{-2}.)

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5. (i) A particle P, of mass $2m$, lies on a rough horizontal table. P is connected by a light inextensible string passing over a smooth pulley fixed at the edge of the table to another particle Q, of mass $5m$, hanging freely. The system is released from rest with the string taut and the hanging part vertical.

If the acceleration of the system is of magnitude $\frac{4g}{7}$, find

(a) the tension in the string, **(3 marks)**

(b) the coefficient of friction between the first particle and the table. **(4 marks)**

(ii) One end of an inextensible string of length 3 m is fastened to a fixed point O 2 m above horizontal ground. Asmall particle is attatched to the other end of the string. The particle describes a horizontal circle 1 m below O. Find, in terms of g, the tension in the string and the angular velocity of the particle . **(6 marks)**

6. A car, of mass 1400 kg, moves with a steady speed of 20 m s^{-1} on a straight rough road with its engine working at a constant rate of 36 kW.

(a) Calculate the resistance R to the motion of the car if the road is horizontal. **(4 marks)**

Given that R is proportional to the square of the speed v of the car,

(b) show that $R = \frac{9v^2}{2}$ **(3 marks)**

When the car is descending a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{20}$, and the resistance to the motion of the car remains unchanged, the speed of the car is 16 m s^{-1} when its acceleration is $\frac{3}{7}$ m s^{-2}.

(c) Calculate the rate at which the engine is working. **(6 marks)**

7. (i) A uniform ladder of weight W and length $2a$ rests in limiting equilibrium with one end on a rough horizontal ground and the other end on a rough vertical wall. The coefficients of friction between the ladder and theground and between the ladder and the wall are μ and λ respectively.

If the ladder makes an angle θ with the ground, where $\tan\theta = \frac{5}{12}$,

(a) show that $5\mu + 6\lambda\mu - 6 = 0$, **(6 marks)**

(b) find the values of μ and λ, given that $\lambda\mu = \frac{1}{2}$. **(2 marks)**

(ii) Show by integration that the centroid of a uniform semi-circular lamina of radius a from the centre is $\frac{4a}{3\pi}$. **(5 marks)**

8. (i) Given that three events A, B and C are such that $P(A) = P(C)$, $P(A \cap C) = \frac{1}{10}$, $P(A \cup C) = \frac{1}{2}$, $P(C|B) = \frac{2}{7}$ and $P(B \cup C) = \frac{4}{5}$, find

(a) $P(A)$, **(3 marks)**

(b) $P(B)$. **(4 marks)**

(ii) The desks in a certain classroom are produced by three different carpenters A, B and C. A produces 30 %, B produces 45 % and the rest are produced by C. It is known that 3 % of the desks produced by A, 6 % of those produced by B and 4 % of those produced by C, are defective. A desk is selected from this classroom at random.

(c) Draw a tree diagram showing all the possible outcomes. **(2 marks)**

(d) Find the probability that the desk is defective. **(2 marks)**

(e) Given that the desk is defective, find the probability that it was produced by C. **(2 marks)**