775 FURTHER MATHEMATICS

1. Introduction

Further Mathematics is designed to prepare students for further studies and applications that are higher than those provided by Advanced Level Mathematics. It examines concepts in pre-calculus, algebraic structures, statistics, mechanics and applications. The style of the examination will continue to change and take into account modern developments.

2. AIMS

In addition to the aims given in the *general introduction* for Advanced Level Mathematics (765/770), the aim of the 775 syllabus is to enable schools to provide an option for:

i. candidates with higher aptitude, ability, and inclinations to study more Mathematics at this level.
ii. candidates who intend to proceed to areas of higher education requiring a deeper understanding and broader knowledge of Mathematics.
iii. candidates who wish to enter job market or into vocational training where a higher demand for mathematics is needed.

3. GENERAL OBJECTIVES

The 775 syllabus should:

a. develop candidates' deeper understanding of mathematical reasoning and processes.
b. develop candidates' ability to relate different areas of Mathematics to one another.
c. provide candidates with a foundation for further study of Mathematics and give them adequate mathematical basis for related disciplines and work at higher level.
d. enable candidates to appreciate the significance of Mathematics to the society in general.

4. ASSESSMENT OBJECTIVES

The objective of the assessment is to test the ability of the candidates to:

a) demonstrate a knowledge and understanding of the principles of the core mathematics topics, mechanics, and simple probability distributions (AO1).
b) apply their knowledge of Mathematics to solve simple problems in mechanics and probability (AO2).
c) apply their knowledge of Mathematics to solve problems for which an immediate method of solution is not available and may involve a knowledge of more than one topic of the syllabus (AO2).
d) select, organised, and use techniques of Pure and Applied Mathematics, as specified in the syllabus, to analyse problems and issues (AO3).
e) interpret problems and write clear and accurate mathematical solutions (AO4).
f) evaluate mathematical statements and theories and justify them through the presentation of clear and systematic proofs (or dis-proofs) (AO5).

5. STRUCTURE OF THE EXAMINATION

5.1 Weighting of the Assessment objectives

Assessment Objectives	Weighting of Assessment Objectives
1. Knowledge and understanding) (AO1).	10 %
2. Application of knowledge (AO2).	20 %
3. Analysis (AO3).	30 %
4. Synthesis (AO4).	30 %
5. Evaluation (AO5).	10 %

5.2 Scheme of Assessment

Paper	Mode of Assessment	Weighting	Number of Questions	Duration
1	Written paper with 50 objective MCQs	20 %	50	$1\frac{1}{2}$ hours
2	Essay questions	50 %	10	3 hours
3	Essay questions	30 %	8	$2\frac{1}{2}$ hours

5.3 Table of Specification (TOS)

Paper Number	Category	Number of questions	Marks	Level of difficulty
1	Knowledge and Comprehension	5	5	* i.e. single star or *one* star
	Application	10	10	*
	Analysis	15	15	Eight (8) of which shall be Single star and 7 double stars (**)
	Synthesis	15	15	Seven (7) of which shall be Single star and **
	Evaluation	5	5	***
	Total	50	50	
2	Knowledge Comprehension	1	10	*
	Application	4	40	two questions (20 marks) of single star (*) strength the rest **
	Analysis Synthesis	4	40	three single * questions (30 marks) and one ** questions
	Evaluation	1	10	***
		10	100	
3	Knowledge Comprehension	1	10	*
	Application	3	40	single star (*) 20 marks strength the rest **
	Analysis Synthesis	3	40	single * 30 marks and 10 marks from ** questions or sections of the questions
	Evaluation	1	10	***

The examination will consist of three written papers. *Questions will be set in S I units.*

Paper one. A multiple-choice paper of one and a half hours carrying one-fifth of the maximum mark. Questions will be based on the entire syllabus; including topics of papers 2 and 3.

Electronic calculators may not be used.

Paper two. A paper of three hours carrying half of the maximum mark will consist of 10 questions of varying length and strength for candidates to attempt **all**.

Paper three. A paper of two and a half hours carrying three tenths of the maximum mark and will consist of eight questions of varying length for candidates to attempt **all**. Questions will not carry equal.

Remarks: For papers two and three, candidates will be expected to have non-programmable electronic calculators and GCE standard booklets of mathematical formulae including statistical formulae and tables.

In papers two and three, candidates are advised to show all the steps in their work, giving their answer at each stage. The value of the acceleration of free fall, g, quoted in paper three will be $9.8\,\text{ms}^{-2}$ except otherwise stated in the question paper.

6. The Syllabus

This syllabus brings together the ‘Modern’ and ‘Traditional’ approaches to Advanced Level Mathematics, the use and notation of set theory will be adopted where appropriate. The Further Mathematics examination will be such that candidates will be expected to cover the entire syllabus. Knowledge of the syllabus for Advanced Level Mathematics (765 and 770) will be assumed and may be tested. Questions will be simple and direct. Complicated and excessive manipulation will not be required. If a numerical answer is required, the question will specify “to do many significant figures or decimal places”, otherwise, the answer may be left in a form such as $\frac{32}{29}$ or $\pi(\sqrt{3}+\sqrt{5})$

TOPIC	NOTES	Objectives or Attainment targets
1. MATHEMATICAL REASONING AND PROOFS		
Meaning of $p \Leftarrow q,\ p \Rightarrow q$ $p \Leftrightarrow q$, Propositions, compound propositions, truth tables, logical equivalence, negation and contrapositive, Qualifiers and quantifiers**.**	The negation and contrapositive of $p \Rightarrow q$.	**Candidates will be assessed on their ability to**: use theorems and mathematical reasoning, techniques to write proofs especially the direct and indirect methods of proofs.
Mathematical proof.	- Direct and indirect proof by deduction.	

Relationship between a theorem and its converse.	-Proofs by induction and contradiction.	

2. FURTHER CONTINUITY OF REAL-VALUED FUNCTIONS

Continuity at a point. Points of discontinuity.	f is continuous at a_o if $\lim_{x \to a_o} f(x) = f(a_o)$. The greatest integer function and other simple examples with emphasis on the geometrical properties of continuous functions are expected. Simple discussion of the continuity of a simple function.	**Candidates will be assessed on their ability to:** Use theorems and notions of continuity for a real valued function to determine points on a set where a function is continuous or discontinuous. Applications for intermediate value theorem shall be tested.
Continuity on an interval.	The theorem that if function f is continuous on an interval I then f (I) is an interval expected without proof.	
The Intermediate Value Theorem for continuous functions and application.	Application for theorem that: If f: [a,b]→ℝ is continuous and $f(a)f(b) < 0$, then ∃ s∈(a,b) where f(s) =0 is expected.	
Sum, product and quotient of continuous functions.		
Continuity of the composite of two functions.		

3. HYPERBOLIC AND INVERSE HYPERBOLIC FUNCTIONS

Properties of hyperbolic and inverse hyperbolic functions.	Properties of the hyperbolic functions will be obtained from the basic definitions $coshx = \frac{(e^x+e^{-x})}{2}$ and $sinhx = \frac{(e^x - e^{-x})}{2}$	**Candidates will be assessed on their ability to:** use the definition of the hyperbolic and equivalent logarithmic forms of the hyperbolic functions to solve problems and functional equations involving the hyperbolic and hyperbolic functions.
Derivatives and integrals of hyperbolic functions.		
Derivatives of inverse hyperbolic functions.		
Logarithmic equivalents of inverse hyperbolic functions.		

4. SEQUENCES AND SERIES

Convergent sequences, bounded sequences, monotone sequences. Inductively defined sequences generated by a simple relation of the form $x_{n+1} = f(x_n)$. Test for	The theorem that every bounded and monotone sequence of real numbers converges is expected without proofs.	**Candidates will be assessed on their ability to:** (i) identify and manipulate convergent series and sequences.

convergence. Divergence of sequences.		(ii) identify divergent sequences and series. (iii) use the sandwich theorem.
Elementary ideas of convergence of a series. The sum of a series as the limit of a sequence of partial sums. Use of method of differences (Telescoping series). Primary test for convergence of series. Further tests for convergence of series. The sandwich theorem.	Knowledge of the behaviour of the p-series $\sum_{r=1}^{\infty}\left(1/r^{p}\right)$ for $p>0$ is expected. Only the comparison and ratio tests will be required.	
Taylor and Maclaurin series. Taylor polynomials, series and applications in evaluating limits of quotients functions.	Derivation and use of the series expansions of e^x, $\cos x$, $\sin x$, $\ln(1+x)$ and other simple expressions will be expected.	(iv) derive power series, expansion for functions and use the series to solve identified problems.

5. FURTHER COMPLEX NUMBERS

De Moivre'sTheorem and its applications.	A geometrical demonstration of a modulus inequality will be accepted. Forms such as $\lvert z_1+z_2\rvert \geq \lvert\lvert z_1\rvert-\lvert z_2\rvert\rvert$ may be required.	**Candidates will be assessed on their ability to:** (i) use DeMoivre's theorem, (ii) work with inequalities in the complex plane, (iii) work out loci in the complex plane, (iv) transformations in complex plane.
Use of the relation $e^{i\theta}=\cos\theta+i\sin\theta$		
Modulus inequalities and applications.		
Loci in the Argand diagram.	Loci such as $\lvert z-a\rvert=b, \lvert z-a\rvert=k\lvert z-b\rvert$ and $\arg(z-a)=\beta$.	
Elementary transformations from the z-plane to the w-plane.	Transformations such as $w=z^2$ and $w=\frac{az+b}{cz+d}$ where a, b, c, d are real numbers may be set.	
Transformation of the plane: geometrical transformations, similarity transformations and their complex number representation of the form z$\mapsto az+b$or z$\mapsto a\bar{z}+b$ where a, b and c are complex numbers and a is	Maps on $\mathbb{R}^2$ which preserve co-linearity and division ratios will be expected.	

not zero, Rigid motion.		

6. FURTHER PARTIAL FRACTIONS

Partial fractions.	Expressions such as $\dfrac{(ax+b)}{(px+q)(rx^2+s)}$.	**Candidates will be assessed on their ability to:** (i) express given rational functions into partial fractions. (ii) Use these partial fractions in solving problems such as the evaluation of integrals and summation of infinite series.
Applications of partial fractions.	Use in the summation of series and integration.	

7. FURTHER INTEGRATION

Motivation and definition of the definite integral.	Geometrical bases of the definite integral as the area under a curve	**Candidates will be assessed on their ability to:** i. Use the concepts learned thus far to compute more complicated integrals. ii. establish reduction formulae for integrals.
Integration using simple substitutions.	The choice and use of simple trigonometric and hyperbolic substitutions for integrands involving quadratic surds is expected.	
Simple reduction formulae.		

8. FURTHER CURVE SKETCHING

Graphs of curves given in Cartesian or parametric form.	Determination of asymptotes including oblique asymptotes is required. Curves such as $y=(ax^2+bx+c)/(px^2+qx+r)$, $x=a(t-\sin t), y=a(1-\cos t)$	**Candidates will be assessed on their ability to:** sketch more complicated functions including curves defined in parametric form.
Interpretation of the sign of f(x,y) in the x-y plane.	Cases such as $(x^2+y^2-a^2)(y^2-4x)<0$ may be set.	**Candidates will be assessed on their ability to:** show their understanding and use of the derivative to determine intervals of concavity for a given function.
Concavity at a point and points of inflexion.		

9. DIFFERENTIAL EQUATIONS

First order differential equations: Origins and geometric interpretations,	The use of the integrating factor $e^{\int P\,dx}$ is expected.	**Candidates will be assessed on their ability to**:

Variable separable First order linear non-homogeneous differential equations of the form $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x. Homogeneous Equations of the form: $\frac{dy}{dx}=f(\frac{y}{x})$ using the substitution y = vx.		i. solve first order, ordinary differential equations using the integrating factor method and the method of separation of variables. ii. transform a given differential equations to obtain equations that can be solved as in (i). iii. solve second order linear, ordinary differential equations with constant coefficients. iv. show understanding of the terms, "particular integral" and complementary function".
Linear second other differential equation $a\frac{d^2y}{dt^2}+b\frac{dy}{dx}+cy=f(x)$, where a, b, c are real constants and a particular integral can be found by trial or by inspection.	The auxiliary equation may have real distinct, equal or complex roots.	
Differential equations reducible to the types above by means of a given substitution.		

10. APPLICATIONS OF THE DEFINITE INTEGRAL

Mean values and root mean square values of a function.		**Candidates will be assessed on their ability to:** i. find mean value of integrable functions. ii. calculate arc length of a curve and areas of surface of revolutions. iii. apply theorems of Pappus.
Arc lengths and areas of surfaces of revolution.	Examples may be set in Cartesian or parametric forms but polar coordinates problems will not be set.	
Theorems of Pappus.		

11. POLAR COORDINATES

Sketching of polar curves.	The convention $r \geq 0$ will be used. Questions may involve finding tangents at the pole.	**Candidates will be assessed on their ability to:** i. sketch polar curves and find tangents and normal to such.
Finding tangents parallel and perpendicular to the initial line.		

12. FURTHER COORDINATE GEOMETRY

Cartesian and parametric equations of a parabola, ellipse and hyperbola.	Questions involving tangents and normal may be set.	**Candidates will be assessed on their ability to:**

The rectangular hyperbola.		manipulate and show understanding and use of properties of the different conic sections and other loci both in parametric and Cartesian forms.
Simple loci problems.		
13. THE VECTOR PRODUCT AND ITS APPLICATIONS		
The vector product $\lvert \boldsymbol{a} \times \boldsymbol{b} \rvert$ and the triple scalar product $(\mathbf{a} \times \mathbf{b}).\mathbf{c}$		**Candidates will be assessed on their ability to handle vectors in $\mathbb{R}^3$ to:** i. find distance from a point to a plane. ii. determine the line of intersection of two planes. iii. find the shortest distance between two skew lines.
Application to areas and volumes.	$\lvert \boldsymbol{a} \times \boldsymbol{b} \rvert$ as area. $(\mathbf{a} \times \mathbf{b}).\mathbf{c}$ as volume.	
Applications to points, lines and planes.	Ability to find distance from a point to a line or plane is expected. Use in finding the equation of a plane.	
14. LINEAR TRANSFORMATIONS		
Finite dimensional vector spaces.	Knowledge of linear dependence, independence, basis and dimensions of vectors in a vector space.	**Candidates will be assessed on their ability to:** articulate the notion of transformation.
Definition and properties of linear transformations. Linear transformation of column vectors in 2 and 3 dimensions.		
Matrix representation of a linear transformation. Composite transformation.	Evaluation of a 3×3 determinant. Singular matrices. Transpose of a matrix.	
The inverse (when it exists) of a given transformation or combination of transformations.	Inverse of a 3×3 matrix. Use in solution of simultaneous equations. Use of the relations $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ and $(\mathbf{AB})^{T} = \mathbf{B}^{T}\mathbf{A}^{T}$	
Image and kernel of a linear transformation.	Kernel is the set of vectors that are mapped to the zero vector.	

15. ALGEBRAIC STRUCTURES.

Algebraic operation in a given set.	Associatively, Closure, identity and inverse elements. Commutativity.	**Candidates will be assessed on their ability to show mastery and understanding of concepts from group theory for example**, (i) problems on construction of groups from binary operations. (ii) establishing (cyclic) properties of groups, etc.
Concept of a group. Axioms of group (G,*). Abelian groups. Properties of a group. Cayley Tables. The groups include: (i) $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{C}$ under addition. (ii) matrices of the same order under addition. (iii) 2×2 invertible matrices under multiplication. (iv) modular arithmetic and addition, modular arithmetic and multiplication. (v) groups of transformations. (vi) symmetries of an equilateral Triangle rectangle and square. (vii) invertible functions under composition of functions, permutations under composition of permutations. Finite and infinite groups. The order of group element, cyclic groups and generators. Lagrange's Theorem	Special emphasis on integers and rational numbers, and groups of symmetries of simple plane figures. Permutation groups, groups of functions, complex numbers, matrices, and integers modulo m. Commutative groups and subgroups. Finite group and divisibility.	
Isomorphism between two groups.	Finite and infinite groups.	

16. DIVISION AND EUCLIDEAN ALGORITHMS:

The greatest Common Divisor and Least Common multiple of integers. Relatively prime numbers and prime numbers. Fundamental theorem of Arithmetic, representation of integers in different basis, Linear Diophantine equations of the form $ax + by = c$, Modular Arithmetic, Linear congruencies.	The theorem $a\|b$ and $a\|c \Rightarrow a\|(bx \pm cy)$ where $x, y \in \mathbb{Z}$., and the division algorithm $a=bq+r$, should be emphasized. Proofs of the fundamental theorem of arithmetic are not required. General solutions of Diophantine equation subject to constraints should be emphasized. Chinese Remainder theorem and Fermat's little theorem are needed.	**Candidates will be assessed on their ability to:** articulate the division algorithms over the integers.

Paper 3

17. MODELLING WITH DIFFERENTIAL EQUATION

Further setting up and solutions of differential equations from simple situations.	No particular knowledge of physical, chemical, biological, economic, etc…., laws are required but mathematical formulation of stated laws is expected. Fitting of initial conditions and discussion of results may be required.	**Candidates will be assessed on their ability to:** apply Mathematics to real life problems. All assumptions needed to formulate a problem will be explicitly specified in each case.
Resisted motion of a particle moving in a straight line. Simple and damped harmonic motion.	Only resisting forces such as $a+bv$, $a+bv^2$ where a and b are constants and v is the speed that will be considered.	

18. NUMERICAL METHODS

Simpson's Rule and its applications.		**Candidates will be assessed on their ability to:** use numerical differentiation (finite difference), techniques and Taylor expansions to approximate the values of definite integrals and solutions of ordinary differential equations.
Numerical solution of the first order and second order, differential equations by step-by-step methods.	The approximations $h\left(\frac{dy}{dx}\right)_n \approx y_{n+1} - y_n$, $2h\left(\frac{dy}{dx}\right)_n \approx y_{n+1} - y_{n-1}$ $h^2\left(\frac{d^2y}{dx^2}\right)_n \approx y_{n+1} - 2y_n + y_{n-1}$ where appropriate will be given but the derivation of these results may be expected.	
Use of the Taylor series method for series solutions for differential equations.		

19. SIMPLE AND DAMPED HARMONIC MOTION

Simple harmonic motion	Any damping will be proportional to the speed of the particle.	**Candidates will be assessed on their ability to**: use differential equations to solve problems in simple and damped harmonic motion.
Dampled harmonic motion		

20. ROTATIONAL DYNAMICS		
Moments of inertia, radii of gyration, including use of the parallel and perpendicular axes theorems.	Proof by integration of standard results given in the booklet of mathematical formulae supplied to candidates may also be required.	**Candidates will be assessed on their ability to:** i. calculate the moment of inertia for rigid body dynamics. ii. calculate the moment of momentum. iii. calculate the energy.(kinetic and potential) for rigid body motion. iv. Solve problems using the principles of conservation of mechanical energy.
Motion of a rigid body under the action of a constant torque.	Questions on connected particles and those involving impulse may be set.	
Moment of momentum about a fixed axis.	Questions may involve conservation of moment of momentum.	
Kinetic energy of a rigid body rotating about a fixed smooth axis.	Questions may involve conservation of energy and the force exerted on the axis.	
Compound pendulum.		

21. APPLICATION OF SCALAR AND VECTOR PRODUCTS		
Vector component of a vector in a given direction.	The moment of a force $\mathbf{F}$ about O is to be defined as $\mathbf{r}\times\mathbf{F}$.	**Candidates will be assessed on their ability to:** i. use vectors algebra to solve problems involving the action of forces on a system of particles.
Work done by a constant force.		
Moment of a force.		
Analysis of simple systems of forces in three dimensions.	A system of forces in three dimensions is either in equilibrium, or can be reduced to a single force, a couple or a couple and a force.	

22. MOTION OF PARTICLE IN TWO DIMENSIONS		
Velocity and acceleration components using Cartesian coordinates.		**Candidates will be assessed on their ability to:** use vectors to manipulate and analyse the motion of particles in Cartesian and polar coordinate systems, and to transform from Cartesian to polar coordinates and vice-versa.
Velocity and acceleration components using polar coordinates.	Derivation of the radial-transverse components of velocity and acceleration is expected. Simple cases of radial-transverse motion may be set but tangent-normal problems will not be set.	

23. OBLIQUE IMPACT OF ELASTIC BODIES		
Impact between two smooth spheres.	Determination of angle of deviation and the kinetic energy lost during impact. Use of the relation $0 \le e \le 1$	**Candidates will be assessed on their ability to:** analyse motions involving direct and indirect collision between moving objects.
Impact between a smooth sphere and a fixed plane.		

24. PROBABILITY DISTRIBUTIONS		
Discrete random variables	Knowledge of the expectation and variance of a function of a discrete random variable.	**Candidates will be assessed on their ability to:** solve problems involving discrete and continuous random variables.
Expectation and variance of a discrete random variable.		
The discrete uniform, binomial, geometric and Poisson distributions.	Knowledge of the expectation and variance for these distributions.	
Continuous random variables.		
Probability density function and the cumulative distribution function.	The use of the relation $F(x_o) = P(X \le x_o) = \int_{-\infty}^{x_o} f(x)\,dx$ is required.	
The expectation, variance and mode of a continuous random variable.	The definition of $E(X)$ and $Var(X)$.	
The uniform and exponential distributions.	The definition of $E(X)$ and $Var(X)$.	
The normal distribution.	Use of the standard normal tables is expected.	
Use of the normal distribution as an approximation to the binomial and Poisson distributions.	Application of continuity correction is expected.	

7. Differences between the 2011 new Syllabus and the old 775 Syllabus

8. SPECIFIC REQUIREMENT FOR THE SUBJECT

a. Good knowledge of the use of non-programmable calculators.

b. TEXT BOOKS AND REFERENCES:

i. Bostock, Chandler, Rourke (1982)
Further Pure Mathematics
StandleyThornes

Leckhampton.

ii. Bostock and Shandler (1985)
Further Mechanics and Probability.
StandleyThornes.
Leckhampton.

iii. Celia, Nice, Eliot (1985)
Advanced Mathematics 3
Macmillan Educational Hampshire.

iv. Brian and Mark Gaulter (2001)
Further Pure Mathematics
Oxford University Press Oxford.

v. Brian J, Tony B. (2001)
Further Mechanics
Oxford University Press Oxford.

vi. Bostock, Chandler (1984)
Mathematics- Mechanics and Probability.
Standley Thornes.
Leckhampton.